Indian Statistical Institute
Computer Science 3
Third Year Students
13May2021
Closed Book Exam – Duration 3 hours.

Total Marks: 85

ANSWER Question 1 to 3 and ONE OF 4 or 5.

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Q1: [Total Marks: 2x8=16]

Fill in the blanks with one or two words or a short phrase so that the sentence is correct and meaningful.

a) The worst case time complexity of Linear search is …………………..

b) AVL is an example of …………………… Binary Search Tree

c) BST searches on an average complete in ……………

d) In a string search operation, if *m* is the length of the pattern, *n* the length of the searchable text the brute force method requires ….. preprocessing time and ….. serach time.

e) For a similar string search using the KMP algorithm, the processing time is ………. and the search time is ……….

f) A binary tree with n nodes has a minimum height of ……….

g) In heap sort of an array of numbers, memory space is saved by thinking of an array as a logical heap by ……………

h) The space complexity of merge sort of an array of n numbers is …………….

Q2. [Total Marks: 3x8=24]

Let A(x) and B(x) be two polynomials of degree n (assume n is a power of 2 throughout this question).

Let A and B be expressed as, using polynomials of degree n/2,

$A(x) = A_0(x)+x^{n/2}A_1(x)$, $B(x) = B_0(x)+x^{n/2}B_1(x)$ using obvious notation.

Then the product of the two polynomials

$AB = A_0(x)\, B_0(x)+x^{n/2}A_1(x)\, B_0(x)+ x^{n/2}A_0(x)\, B_1(x) + x^{n}A_1(x)\, B_1(x)$.

Assume that the polynomials are stored as arrays containing non zero coefficients and indices, as is standard.

Let T(n) be the time complexity of computing the product of the two polynomials of degree n.

a.) Develop a recurrence relation for T(n) GIVE DETAILED EXPLANATIONS OF what contributes to T(n) at each step of the above computation

b.) Show that T(n) = O(n^2)..

c.) Explain, without any detailed calculation, how the time complexity can be reduced by a modification of the above algorithm. Your Explanation must be detailed enough to be able to write a pseudo code though you do not have to write the pseudo code.

Q3.[Total Marks: 5x5=25]

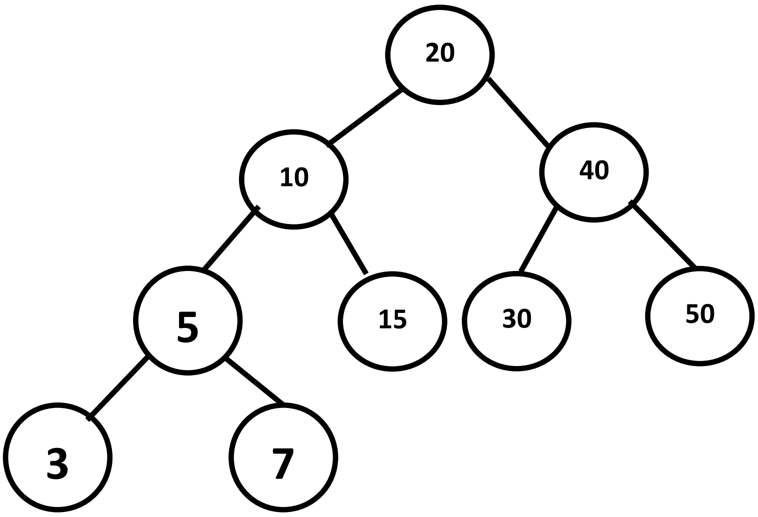
a.) Given a binary tree, write a C function that takes as parameter the pointer to the root node of a binary tree and prints out the in order traversal of the tree.

b.) Reconstruct a binary tree from the following traversals. Show all the steps with explanations.

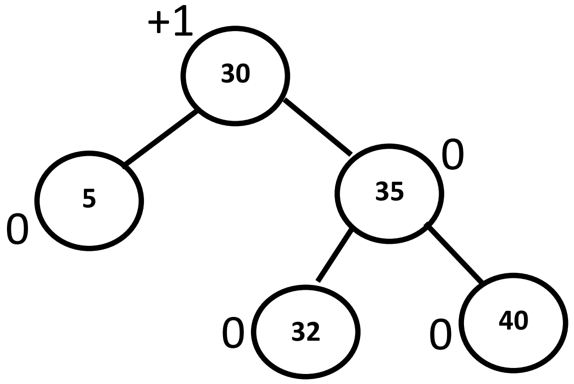
Postoder: T Q S D E A M C F R X P

Inoder: T S Q A E D P M X C R F

c.) Determine if the following tree is AVL or not. Show detailed labelling of the tree with Balance Factors of each node



d.) Insert the number 45 in the following AVL tree. Resulting tree must be AVL. Please explain each step and show the state of the tree in all intermediate states, including Balance Factors as appropriate.



e.) Prove that the worst case AVL trees with n nodes have height h = O(log n)

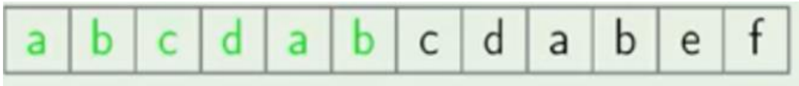
Q4.[Total Marks: 5+5+10=20]

For a string S of length n the prefix function is defined as an array π of length n, where π[i] is the length of the longest proper prefix of the substring S[0…i] which is also a suffix of this substring. A proper prefix of a string is a prefix that is not equal to the string itself. By definition, π[0]=0.

a.) Calculate the pi function of the string "ababababca"

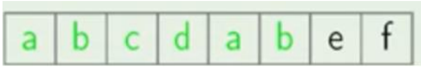
b.) Prove that π[i+1] cannot be greater than π[i]+1.

c.) Suppose we have a Text String:



a	b	c	d	a	b	c	d	a	b	e	f

And a Search string:



a	b	c	d	a	b	e	f

Explain how the search is done. You can use the pi function of the search string

Pi[i]= 0 0 0 0 1 2

Q5.[Total Marks: 5x4=20]

a.) Explain the concept and the purpose of Dictionary Data structure and its representation in a hash table using any simple example of your choice.

b.) Explain the concept of Universal Hashing Functions in terms of their properties.

c.) Give an example (without proof) of an actual construction of a set of universal hashing functions.

d.) Explain how Universal hashing functions can help in reducing collision in a hash table.